Maxwell distribution of molecular kinetic energies:-

The fraction of molecules having kinetic energies in the range of ϵ and $\epsilon + d\epsilon$ viz $\frac{dN_\epsilon}{N}$ can be determined as follows:-

$$\epsilon = \frac{1}{2} mc^2 \quad \text{—} \ (1)$$

$$c^2 = \frac{2\epsilon}{m} \quad \text{—} \ (2)$$

$$2c\,dc = \left(\frac{2}{m}\right) d\epsilon$$

or

$$c\,dc = \frac{d\epsilon}{m} \quad \text{—} \ (3)$$

Then,

$$c^2\,dc = c\,\frac{d\epsilon}{m}$$

$$= \left(\frac{2\epsilon}{m}\right)^{1/2} \left(\frac{d\epsilon}{m}\right)$$

$$\boxed{c^2\,dc = \frac{\sqrt{2\epsilon}}{(m)^{3/2}}\,d\epsilon} \quad \text{—} \ (4)$$

Replacing $c^2\,dc$ using equation for Maxwell molecular velocity distribution, we have

$$p(c)dc = \frac{dN_\epsilon}{N} = 4\pi\left(\frac{m}{2kT\pi}\right)^{3/2}\left(\frac{\sqrt{2\epsilon}}{m^{3/2}}\right) d\epsilon \exp\left(-\frac{\epsilon}{kT}\right)$$

$$\boxed{p(c)dc = \frac{2\sqrt{\epsilon}}{\sqrt{\pi}\,(kT)^{3/2}} \exp\left(-\frac{\epsilon}{kT}\right) d\epsilon} \quad \text{—} \ (5)$$

Maxwell distribution of molecular kinetic energy

Effect of temperature in distribution of molecular velocities :-

The most probable velocity increase with rise in temperature as mentioned in the previous plot. There is a shifting in the entire curve.

The rise in temperature increases the number of molecules having high velocities.

According to equation

$$p(c)\,dc = 4\pi\left(\frac{m}{2kT\pi}\right)^{3/2} c^2 \exp\left(-\frac{mc^2}{2kT}\right) dc \quad — (1)$$

The exponent has negative sign and temperature T in the denominator. The factor that increases in temperature. This is known as Boltzmann factor.

since $\frac{1}{2}mc^2 = K.E = \epsilon \quad — (2)$

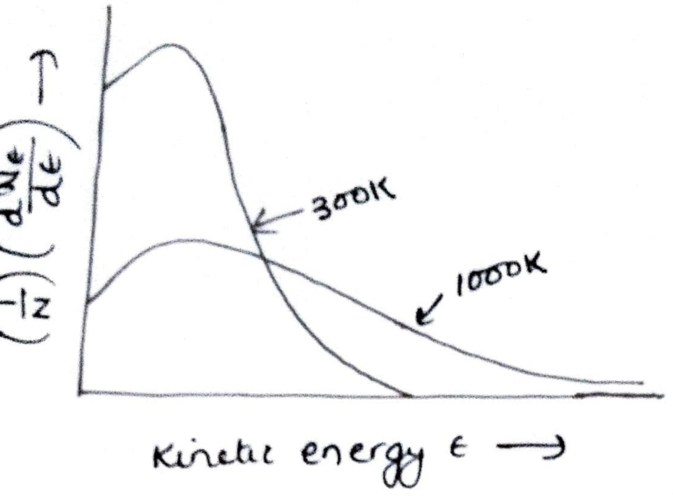
$$\exp\left(-\frac{mc^2}{2kT}\right) = \exp\left(-\frac{\epsilon}{kT}\right) \quad — (3)$$

ϵ = K.E. of the molecule of the gas.

The greater the temperature, the greater is the value of ϵ. Hence there is increase of Boltzmann factor with the increase in temperature.

This conclusion finds application in the theory of reaction rates.

The Maxwell distribution of kinetic energy at two different temperature can be represented as



According to the plot the probable K.E. increases with the increase in temperature.

The maximum in the probability function corresponds to the most probable kinetic energy.

From the equation (5)

(i) The most probable kinetic energy is given by $kT/2$ per molecule or $\frac{RT}{2}$ per mole of the gas

(ii) The average kinetic energy per mole is given by $\left(\frac{3}{2}\right) N_A kT = \left(\frac{3}{2}\right) RT.$

Above two relations are in agreement with that obtained from the kinetic theory.